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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

PLANCK'S RADIATION FORMULA AND THE CLASSICAL ELECTRODYNAMICS.

By David L. Webster.

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PLANCK'S RADIATION FORMULA AND THE CLASSICAL ELECTRODYNAMICS.

By David L. Webster.

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Introduction. As Poincare [1] has pointed out, Planck's formula for black body radiation, or any other formula giving only a finite amount of energy per unit volume in the radiation field, involves the assumption of some discontinuities in the process of absorption and emission. Thus Planck assumes that while an oscillator may absorb energy continuously, it may radiate only when the energy is an integral multiple of $h\nu$, and that if it does then radiate, all its energy is radiated at once.

This assumption, being inconsistent with the classical electrodynamics, involves the abandonment of the explanations that the classical system gives of many phenomena. Its explanation of inertia, for example, depends directly on the theorem that every part of an accelerated electron will radiate electric forces proportional to its acceleration, and that these forces, acting on other parts of the electron, produce the force of inertia, proportional and opposite to the acceleration. This explanation must be abandoned if we make Planck's assumptions, which deny the existence of these forces in most cases.

Likewise, according to the retarded potential theorem, the classical explanations of all phenomena of the propagation of light through matter may be put entirely on the basis of continuous re-radiation from vibrating electrons, whose energy in many cases never reaches the value $h\nu$, because the amplitude of a forced vibration is so small unless the frequency is near that of resonance. Although light phenomena other than scattering are not ordinarily treated in this way, any method derived from the classical field equations must necessarily be equivalent to any other, and any assumptions that make one of these methods give wrong results must necessarily do the same for all. Therefore, if Planck's assumptions are true, all such explanations must be abandoned, and we must create a whole new theory of optics.

1 Journal de Physique, (5) **2**, p. 1 (1912).

It might be supposed that we could remedy this defect by assuming the existence of two classes of oscillators, the first obeying the classical laws and the second obeying Planck's assumptions, and in that way account for both optical and thermal phenomena. This hypothesis, however, does not seem tenable.

For although the classical oscillators would do their best to account for other light phenomena, they would tend to give Rayleigh and Jeans's radiation law (which any system obeying the classical dynamics must give) rather than Planck's. Or, in terms of Planck's derivation, their entropy, computed from the thermodynamic probability, would be different from that of his oscillators because their distribution on the energy diagram would be different. Hence the radiation law would be different also, if any noticeable percentage of these oscillators were present. Moreover, Jeans's law, which they would tend to make the radiation obey, gives an intensity whose ratio to Planck's becomes infinite rapidly as λT diminishes. Thus an extremely small percentage of classical oscillators would give large deviations from Planck's law, while the Planck oscillators would not give the well known relations between absorption and dispersion, and other optical laws that are explained by the classical theory.

Moreover, unless the percentage of each class of oscillators for each frequency were exactly the same for every substance at each temperature the distribution of energy in the spectrum of a cavity would depend on the substances contained in its walls. Such a result is well known to be contrary to the second law of thermodynamics.

As Planck [2] says, the assumptions he makes are not the only ones that can lead to his law of radiation. The object of the present investigation is therefore to see whether the abandonment of the classical electrodynamics and its explanations of these and other phenomena is really necessary, and if it is not so, to find a mechanism giving both heat radiation and optical phenomena, and at the same time consistent with other phenomena as far as possible. The conclusion is that it is not necessary, and that the formula can equally well be derived from other assumptions, inconsistent with the classical mechanics only as applied to the internal structure of the electron, and consistent with the classical electrodynamics and its explanations of many phenomena; and, moreover, this explanation is based on a mechanism which has been shown by Parson [3] to be very useful in

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2 "Heat Radiation," English translation by Masius, 154 (1914).
3 Not yet published.

explaining chemical affinities, and which has other advantages over Planck's oscillator and most other models of the quantum.

Planck's Assumptions. The basis of the derivation of Planck's formula is Boltzmann's relation between entropy and probably, which Planck puts in the form $S = k \log W$ where S is the entropy and k is the gas constant reckoned for a single molecule, and W is the "probability," considered as the number of ways in which the system can be arranged so as to be indistinguishable from its present form.

In applying this law to a system of similar molecular oscillators, two arrangements are considered indistinguishable if they give equal numbers of oscillators in each of certain groups, any group, the n^{th}, being defined by the condition that the energy of every oscillator in it must lie between $(n-1)\, h\, \nu$, and $n\, h\, \nu$. For this rule to be a real criterion of indistinguishability, we must have the density of their representative points of the energy scale constant through each one of these intervals and changing abruptly on going from one interval to the next.

These assumptions, as Planck shows, give for the mean energy of the oscillators in the n^{th} group the value $\left(n - \frac{1}{2}\right) h\, \nu$. By setting the entropy equal to its maximum value consistent with a given total energy, he finds for the probability that a given oscillator will be in the n^{th} group the value $w_n = \alpha\gamma^n$ where $\alpha = \dfrac{2\, N\, h\, \nu}{2\, E - N\, h\, \nu}$ and $\gamma = \dfrac{2\, E - N\, h\, \nu}{2\, E + N\, h\, \nu}$, E being the total energy of N oscillators.[4]

Now to obtain a law of distribution of energy in the black body spectrum, it is necessary to have some law of emission and absorption for the oscillators. To keep in touch with the classical electrodynamics, he assumes that an oscillator absorbs energy as that system would require if one might neglect reradiation entirely; but to obtain the discontinuities required, he assumed, not the classical law of emission, but that the oscillator can emit only when the representative point reaches the boundary of one of these intervals, and that, if it does emit, it must emit its whole energy, $n\, h\, \nu$.

With these assumptions, he proves that the energy of such an oscillator, exposed to radiation of continuously distributed frequencies, will increase at a constant rate, thus insuring the constancy of the density of points in each interval; and he obtains the proper reduction of their density from one interval to the next by assuming that the

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[4] Planck, l. c., § 139.

probability η that the oscillator will emit when its energy reaches the value $n\,h\,\nu$ is given by the formula $\frac{1-\eta}{\eta} = pI$, where p is a constant, and I is the intensity of the electric vibration per unit interval of frequency. That is, the stronger the light, the more likely the oscillator is to go on accumulating energy. The value of p is obtained by setting the mean energy thus found approximately equal, for extremely large values of I, to the value it would have if the oscillator radiated as demanded by the classical theory. This is a new assumption, independent of the previous ones, and based on the experimental fact the Rayleigh's law of radiation is true for large values of λT.

The next step is to identify the distribution of points thus found with that obtained above from entropy considerations, and thus find I in terms of ν and T, and from I the energy density per unit frequency interval,

$$u = \frac{8\,\pi\,h\,\nu^3}{c^3}\,\frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

Assumptions of the Present Theory. The starting point of the present theory is Parson's magneton, a ring of negative electricity of a diameter perhaps $\frac{1}{10}$ that of a hydrogen atom, revolving on its axis with a velocity of the order of that of light. This has been proposed by Parson as a substitute for the classical electron, and has given good results in the explanation of chemical affinities.

These magnetons are supposed by Parson to be free to move in a sphere of continuously distributed positive electricity, in which, as he shows, they have a strong tendency to group themselves in eights, thus giving the foundation of the periodic table of the elements. A detailed discussion of their groupings and the surprising way in which they explain not only the table, but also the exceptions to its rules, and many other chemical phenomena, will be found in his paper.

Inquiring into what may be expected of the vibrations of the magneton, we find a state of affairs somewhat more complex than in the classical electron theory. For we have not only the attraction of the positive electricity through which it moves, and the electrostatic repulsions of the other magnetons, both tending to make its equilibrium stable, but also the magnetic attractions of the others, tending to make it less stable, and perhaps still other repulsions by them, proportional to some inverse power of the distance higher than the second, and therefore having a strong tendency to promote stability. The combined effect must, of course, be a stable equilibrium.

Moreover, as possible modes of vibration, we have not only a rigid displacement of the magneton as a whole, but also a disturbance of the flow of electricity around it, that may give electrical oscillations. These may be thought of as superposed on the continuous flow just as they would be in the case of a large ring of wire, heavily charged with negative electricity and at the same time carrying a current around the ring and performing electrical oscillations which may displace the centre of charge to any point in its plane. The displacement of the center of mass of the magneton during these oscillations might be anything, depending on how the charge was distributed and on what changes of thickness of the ring might be caused by the changes in distribution of its charge. We shall assume here, purely for convenience, that the center of charge and the center of mass move together. We shall also assume that any number of these waves may be superposed without disturbing one another, except by electrostatic action, as in the case of the ring of wire.

Let us now consider a magneton whose geometric center is displaced in the plane of the ring by a distance ξ', and whose center of charge and mass is displaced relative to the ring by a distance ξ'', or in all a distance $\xi = \xi' + \xi''$. We shall divide the intra-atomic forces acting on it into the following five classes:

(1) The resultant of the attraction of the positive electricity through which it moves and the repulsions of the other magnetons, equal to $-f\xi$, and acting on the electricity itself, rather than on the rest of the structure;

(2) The resultant of the magnetic attractions and all non-electrostatic repulsions between them, equal to $-f'\xi'$ and applied to the structure of the ring;

(3) The internal forces of the magneton, giving a force $-f''\xi''$ on the electricity and $+f''\xi''$ on the ring;

(4) The force of inertia, $-\,m\dfrac{d^2\xi}{dt^2}$ due to radiations caused by the acceleration of the electricity, and acting on it;

(5) The damping force $+\dfrac{2}{3}\dfrac{e^2}{c^3}\dfrac{d^3\xi}{dt^3}$ due to radiation, equal for simple harmonic motions depending on $\sin \omega t$ to $-g\dfrac{d\xi}{dt}$ where $g = \dfrac{2}{3}\dfrac{e^2}{c^3}\omega^2$.

(6) Another damping force $-g''\dfrac{d\xi''}{dt}$ due to an assumed tendency

of the internal mechanism of the magneton to transfer energy from the oscillation of the electricity on it to the steady current around it.

f, f', and f'' are assumed to be constant, and we shall assume the equilibrium of the ring under the action of (2) to be so stable that for visible and ultraviolet frequencies, the motion of the center of charge is due chiefly to the oscillation of the electricity on the ring, and very little to the motion of the ring as a whole. In the infra-red, the motion of the atom itself will, of course, change the whole character of the oscillation.

This plan does not agree entirely with Parson's assumptions, which make the magnetons interchange their positions in the atom rather freely; but I doubt if this is a serious difficulty, since even with these assumptions, a strong collision, with its accompanying distortion of the positive sphere, might easily cause considerable changes in their arrangement.

The damping force (6) is contrary to the laws of the classical mechanics as applied to the internal structure of the magneton. But this conflicts with no experimental facts, and, as we have seen, we must expect a violation of these laws somewhere in the absorbing and emitting system. The mechanism of this transfer cannot be ordinary electromagnetic induction, because simple considerations of symmetry show that the mutual inductance between these oscillations and the steady current is zero.

The energy transferred from the oscillations, according to these assumptions and others to be made below, will be found very small compared to the whole magnetic energy of the magneton, which is of the order of magnitude of mc^2. (This value will be discussed in more detail in a subsequent paper showing why we cannot expect the magnetic properties of the magneton to be detected by experiments on cathode rays or photo-electrons.) Therefore the increase in the velocity of the steady current that is given by this energy must be very small compared with the original velocity, and so will not interfere with the explanation of chemical phenomena by the magnetons.

Neglecting the electrostatic influence of neighboring molecules, we may say that all the above forces on the electricity must balance, and so must all those on the ring. This gives the equations,

$$(1) \qquad -f\,\xi - f''\,\xi'' - m\,\ddot{\xi} - g\,\dot{\xi} - g''\,\dot{\xi}'' + e\,E_\xi = 0$$

$$(2) \qquad -f'\,\xi' + f''\,\xi'' = 0.$$

Combining these, we obtain

$$(3) \qquad m\ddot{\xi} + \left\{ g + \frac{g''f'}{f'+f''} \right\} \dot{\xi} + \left\{ f + \frac{f'f''}{f'+f''} \right\} \xi = e\,E_\xi$$

This equation, being of the standard form for forced harmonic motion with damping, shows at once that the classical theory of the propagation of light through matter is reproducible from this model. For, if the electric force is inclined obliquely to the plane of the magneton, the component in the direction perpendicular to this plane will produce practically no effect, on account of the immobility of the charge of the magneton in this direction. Other magnetons, being tipped in other directions, will supply the mobility that is lacking in the one we have considered.

In the infra-red, where the vibrations of atoms as a whole begin to be of importance, we must superpose the motion of the sort considered above on that of the atom itself. Since each atom has in general very little magnetic moment, it must have magnetons (especially in the groups of eight) turned in all directions. Therefore some of them will always be vibrating with a motion of the center of charge relative to the ring. Their rates of absorption, however, cannot be found from this equation because in a solid or liquid the vibrations of the atoms as a whole will be governed chiefly by collisions with other molecules, while in a liquid or gas the rotation of the molecules will change the direction of the axis of vibration continually and thus prevent the accumulation of large amplitudes. Therefore the rates of absorption are much less than this equation would indicate, as soon as we get to frequencies such influences become noticeable. That this occurs to some extent even in the visible spectrum at ordinary temperatures is indicated by the readiness with which absorption of most visible light generally produces heat rather than other effects, such as photo-electric currents or chemical changes, and also by the well known widening of absorption lines by pressure. The fact that such influences may result in a transfer of a certain amount of energy from the vibrations to molecular motions rather than to the steady current of the magnetons, is, as we shall see, entirely immaterial for the derivation of Planck's law, since this derivation depends on considerations of probability that are unchanged by this transfer.

Since each high frequency magneton exposed to radiation will execute a steady vibration, it must store up energy at a constant rate. Likewise at low frequencies, since the oscillations themselves are independent of the amount already stored, however much they may be affected by molecular motions, the rate of storing will, on the average,

be constant. This result, which is attained in Planck's theory by the prohibition of all re-radiation or external influences, is quite necessary for the derivation of his law.

This accumulation of energy, moreover, must continue until the energy stored in the oscillator reaches some integral multiple of $h\nu$, so that even at the shortest wave lengths or lowest temperatures it must always be able to attain the value $h\nu$ at least. At longer wave lengths or higher temperatures, the oscillator will accumulate many quanta. It is, therefore, important to see how this energy will affect its properties.

If the oscillator is the classical electron,[5] the energy must all be stored in its vibration, so that for an amplitude ξ_0 to hold a quantum, we must have the energy

$$\frac{1}{2} m \omega_0^2 \xi_0^2 = h\nu = \frac{h\omega_0}{2\pi},$$

or

$$\xi_0 = \sqrt{\frac{h\lambda}{2\pi^2 m c}}$$

This value, being proportional to $\sqrt{\lambda}$, will be greatest for the longest waves for which the oscillator may safely be assumed to be an electron in a non-vibrating atom. Since the values of the Zeeman separations lead to this assumption certainly throughout the visible spectrum, we may apply this forumla for $\lambda = 8000$ Å finding

$$\xi_0 = 3.1 \text{ Å}.$$

Comparing this with the distance between carbon atoms in diamond, found by W. H. and W. L. Bragg [6] to be 1.52 Å, we find that the length of the whole vibration would have to be over four times the diameter of the atom.

This is another serious difficulty in the way of Planck's theory. For even if the oscillation consists of all the electrons in the atom moving together, so that the amplitude required is reduced in proportion to the square root of their number, the distance which the group may go before some of them get outside the atom is also greatly reduced. Moreover if the positive sphere is not absolutely uniform in density, the distance they can go before the frequency of vibration is changed is even less.

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5 Planck does not specify the charge or mass of the oscillator: this paragraph therefore starts with the most plausible assumption as to its nature.

6 Nature **91**, 557 (1913).

If one could assume without conflict with measurements of the Zeeman effect that the atom itself takes an appreciable part in the motion at such frequencies, then the amplitude required for this energy would be less than what we have found. In this case, however, the electron would have to be able to distinguish carefully between the part of the atom's energy that it could include in the quantum and the part due to heat motions and vibrations with other frequencies.

There is, furthermore, another difficulty caused by the fact that the transfer of energy to heat motion of the molecules would increase rapidly with the amplitude of the oscillator, so that the rate of increase of the vibratory energy would diminish, rather than remain constant as it must for Planck's law.

These difficulties are all avoided by the magneton, because of its storing its energy in its steady current, and thus making the oscillation, as we have noted above, independent of the amount already stored. The smallness of the increase of the steady current that is required for this is evident from the fact that mc^2, which gives the order of magnitude of the magnetic energy, is 7.9×10^{-7} erg, while even at 1000Å, where the oscillator practically never holds more than one quantum, the quantum is only 2×10^{-11} erg.

Another point that is as necessary for the derivation of Planck's law as a satisfactory method of absorbing and storing the energy is a satisfactory law of emission. For this Planck assumes p. 153, "that the emission does not take place continuously, as does the absorption, but that it occurs only at certain definite times, suddenly, in pulses, and in particular we assume that the oscillator can emit only at the moment when its energy of vibration, U, is an integral multiple n of the quantum of energy, $\epsilon = h\nu$."

Just how a sudden pulse can have a definite frequency is difficult to imagine, and is not stated in his book.

The experimental facts, moreover, are against the assumption that the emission is absolutely instantaneous. For Fabry and Buisson,[7] have found that the path difference for interference observable in spectrum lines from the inert gases at low temperatures is often a very considerable fraction of a meter, and in the case of Krypton, with the tube immersed in liquid air, the path difference for wave length 5576 Å is 53 cm. or 950000 wave lengths. These path differences, moreover, are given within the limits of experimental error by Schönrock's formula derived from the kinetic theory, on the assumption that the light of each oscillator is really monochromatic and that the width of

[7] C. R. **154**, 1224–7 (1912), or J. de Phys. **2**, 5, 442–64 (1912).

the line is due only to the Doppler effect. Thus we may conclude that in such cases, at least, the time required for an emission is really quite considerable, and that Planck's assumption of practically instantaneous emission must be abandoned.

One way to evade this difficulty in the derivation of Planck's law is to assume that the radiation takes place at a constant rate until the whole energy, $n h \nu$, is emitted. This, however, is easier said than done; and one would be strongly tempted to look for some other radiation law, if Planck's had not been so well confirmed by direct experiments, not only those quoted in his book (p. 169) but also those published more recently by Coblentz,[8] and indirectly by the appearance of his constant h is the laws of so many other phenomena, such as specific heats and photo-electric effects.

This constant rate of radiation cannot be obtained by making the effect of the vibrating charge on the ether suddenly become some constant multiple of that which the classical theory would give, because it is well known that any such law would make the amplitude die out exponentially, rather than linearly, and approach a finite value, rather than zero. For the same reason, it cannot be obtained by having the absorbed energy stored as potential energy of an electron transferred from one equilibrium position in the atom to another, and re-emitted when the electron is jarred out of the latter position and falls back, with oscillations, into the former. Such models, moreover, would also be open to the objection to Planck's oscillator, that, to give a single quantum, they require too great an amplitude of vibration.

Another serious difficulty for such models of the oscillator is the fact that an essential point in the derivation of the law is the assumption that an oscillator may acquire an amount of energy that is *any integral multiple* of the quantum before it radiates. This applies, for example, to Bohr's atom, in which the transition from one equilibrium position to another makes the electron give out just one quantum, and the transitions between all other such positions will give different frequencies.

Still another point where such models are apt to be insufficient is in the explanation of photo-electric phenomena. For if, in the "lower" equilibrium, the electron is in a region of positive potential, and in the "higher" one it is either in such a region or removed to infinity, then, when it escapes from the higher position, it will either not leave the atom at all or else leave with an infinitesimal velocity. The only way

[8] Bull. Bur. Stan., Jan. 15 (1914).

for it to obtain the kinetic energy $h\nu - W_0$ indicated by experiments such as those of Richardson and Compton [9] is to have the higher position in a region of negative electrostatic potential, that is, to have the electron vibrate toward a mass of negative electricity without being thrown to one side of the path by the repulsion. This seems distinctly difficult to accomplish, and I do not know that it has ever been done.

To account for Planck's law and other experimental results, therefore, we are driven to the unpredictable and unsatisfactory assumption that when the energy stored in the magneton reaches any integral multiple of $h\nu$, the internal mechanism of the ring may start another oscillation, larger than the absorbing one, and that this emitting oscillation will maintain a constant amplitude, deriving its energy in some way from the steady current, until the excess energy stored in the magneton has been radiated, and its total energy is reduced to a standard amount. The probability η of starting to radiate at any particular multiple of $h\nu$ is the same as in Planck's theory.

To be sure that this mechanism is not, like Planck's, too big for the atom, we may calculate the amplitude of the emitting oscillation that is required to emit the energy faster than it is absorbed.

Rewriting equation (3) in the more condensed form

$$(4) \qquad m\ddot{\xi} + b\dot{\xi} + \kappa\xi = eE_\xi = eE_0 \cos \omega t$$

where b and κ are abbreviations for the coefficients in (3), we may evaluate the rate of absorption for the frequency $\nu = \dfrac{\omega}{2\pi}$ as

$$(5) \qquad R_\nu = e\overline{E_\xi\dot{\xi}}.$$

In using this equation (5) we are implicitly assuming that all the work done by the electric force goes into energy stored by the magneton, and we are therefore neglecting its extremely small re-radiation. Solving (4) for the case of a steady vibration, and substituting in (5), we obtain

$$R_\nu = \frac{\frac{1}{2}e^2 E_0^2 \omega_0^2 b}{m^2(\omega_0^2 - \omega^2)^2 + b^2\omega^2}$$

where $\omega_0 = \sqrt{\dfrac{\kappa}{m}}$

We may replace E_0^2 now by $Id\nu$ and integrate with respect to ν to obtain the rate of storing energy by the magneton. In this integra-

[9] Phil. Mag. **24**, 575 (1912).

tion, ω may be replaced by ω_0 except in the expression $(\omega_0^2 - \omega^2)$ [10]. Performing the integration in this way, one obtains for the rate of absorption by vibrations in the ξ direction only, the value

$$\dot{U} = \frac{e^2}{4\,m} I \tag{6}$$

independent of ω_0 or b.

To see what amplitude is required to emit as fast as this, we may use the well known equation for the rate of emission,

$$\tfrac{1}{3}\frac{e^2}{c^3}\,\omega_0^4\,\xi_0^2.$$

For the limiting case, let us set this equal to $\dot{U}$ and thus find the least permissible amplitude, ξ_0.
Thus

$$\xi_0^2 = \frac{3\,c^3\,I}{4\,m\,\omega_0^4}$$

Inserting the value $\omega_0 = 2\pi\nu$ and

$$I = \frac{32\,\pi^2\,h\,\nu^3}{3\,c^3}\,\frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

we have

$$\xi_0 = \sqrt{\frac{h}{2\,\pi^2\,m\,\nu}\,\frac{1}{e^{\frac{h\nu}{kT}} - 1}} \tag{6}$$

Evidently this amplitude will be greatest for the lowest frequencies for which equation (3) can be expected to hold with any accuracy, that is for frequencies situated somewhere in the visible range. It is, of course, impossible to set any precise limits here, but for the purpose of forming a rough idea of these magnitudes, let us calculate the amplitude for $\lambda = 6000$Å at temperatures of 300, 1000, and 2000° absolute.

For this case we have $\nu = 5 \times 10^{14}\ \text{sec}^{-1}$ and since $h = 6.415 \times 10^{-27}$ erg. sec, $m = 8.8 \times 10^{-28}$ gm and $k = 1.34 \times 10^{-16}\ \frac{\text{erg}}{\text{deg}}$ we have $h\nu = 3.2 \times 10^{-12}$ erg, while $k\,T$ has the values 4.02×10^{-14}, 1.34×10^{-13}, and 2.68×10^{-13} erg respectively. Thus the values of ξ_0 at these temperatures are 3×10^{-17}, 2.5×10^{-5} and 6.5×10^{-2} Å respectively.

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10 See Lorentz "Theory of Electrons," note 62.

If now the radius of the magneton is about 10^{-9} cm, or 0.1Å, the first two of these values are well within the limits of the possible amplitude of vibration of the center of charge, while the last is comparable with the radius of the magneton, and therefore suggests the possibility that the disturbance might result in the ejection of the magneton as a thermion. From the form of the expression for ξ_0 one would be led to infer that at lower ν's this would happen at lower temperatures. We must remember, however, that such speculations are very untrustworthy, because this formula (6) is developed on the hypothesis that the vibration is not disturbed by molecular motions — a hypothesis which we have seen to be incorrect even at the 6000 Å, which we have just considered. Hence we may infer that the amplitudes required here are probably within the possibilities of this mechanism, and that the results noted above point to a qualitative explanation of the emission of electrons by hot bodies.

A priori, this mechanism does not seem so good as some scheme for obtaining the quantum from the dimensions and charges of different parts of an atom. Its justification, however, lies in its ability to correlate experimental facts which are inconsistent with these other schemes.

First, by means of this mechanism, we may account for Planck's law of radiation, which, as we have seen, is not given by atomic models in which the atom can hold only a definite number of quanta.

Second, these assumptions do not, like Planck's, appear to be inconsistent with observed path differences in interference experiments, or known magnitudes of atoms.

Third, the magneton has been found by Parson [11] to be very efficient in explaining not only the periodic table of the elements, but even many of the exceptions to the laws indicated by this table.

Fourth, since it makes the emission of a quantum take the form of a very energetic oscillation, it gives a good chance for thermal emission of corpuscles, and, at low frequencies, for the interaction between colliding molecules that is necessary for the interchange of energy between heat and light, or in the absence of collisions for the flourescence of very rarified gases.[12] Moreover, since the amplitude is not dependent on certain equilibrium positions, as in most atomic models giving quanta, these assumptions can give an account of the photoelectric and photo-chemical phenomena at higher frequencies.

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11 l. c.

12 For a discussion of this point, see Phys. Rev. N. S. **4**, 177–94 (1914).

Finally, this scheme gives Planck's formula without violating the classical laws determining the electromagnetic field in terms of the motions of electricity. Therefore it can also give an account of many optical phenomena, such as those of the propagation of light through matter, for which Planck's assumptions give no account at all.

Thus this theory correlates the phenomena of heat radiation, chemical affinities, thermions, photo-electric action, interference and many other optical effects, without the use of a mechanism that is inconsistent with known magnitudes. Consequently it may be pardoned for requiring these few arbitrary assumptions.

With regard to the internal mechanism that produces the emitting oscillations we can say very little, except that our intuition would naturally lead us to suppose that they could not exist. But we are here considering things of an order of magnitude so small that, while imagination is as valuable as ever, our intuition is very unreliable. For, after all, when we predict what a mechanism will or will not do, our predictions are not based ultimately on any ability of our own minds to guess the truth of a question, but rather on the accumulated results of our everyday experience, reenforced perhaps by that of our ancestors, and made more accurate by the results of premeditated experiments in the laboratory. This experience is all with things of visible size, and our intuitions tell us most naturally and emphatically what any mechanism would do if it obeyed the laws of visible things.

But why should this mechanism obey these laws? An enormous giant, capable of seeing only celestial objects and their motions, would predict everything in terms of inverse square laws of acceleration, and would be very much surprised, on increasing his power of vision, to find how rarely terrestrial objects obeyed such laws. Forces, especially those of friction, would then compel his attention; and they in turn would lay the foundation for another shock when he came to study the kinetic theory, with its frictionless attracting and repelling molecules, and their account of the origin of friction and other forces between larger bodies. Here he would have the sympathy of all students, whose intuition tells them that a roomful of ultramicroscopic bouncing billiard balls would bounce slower and slower until they settled in a hopeless heap on the floor. In another step, to the electron theory, the character of the forces changes again, and the phenomenon of inertia, the foundation of the dynamics of larger bodies, is explained as a result of electromagnetic forces. Is it, then, at all surprising if still more of these radical changes are in store for us in studying the ether and the internal mechanism of the magneton?

We have been guided thus far by the necessity of correlating experimental facts. Let us therefore judge these assumptions by the number of experimental facts for which they account.

It would undoubtedly be possible to set up numerous models and sets of equations which would account, in a more or less complex way, for this law of emission, but it would not be profitable at present, as the laws would not be simple, and probably not very instructive. Therefore we shall not attempt to do so here.

The Derivation of Planck's Law. Planck's formula may now be derived by an argument almost like his, except as follows:

First, for the proof, Part IV, Chapter II, that the rate of absorption is constant with respect to time and proportional to the intensity of the light, we may substitute the fact that this is a well known result of an equation of the type of (3) when the intensity of the light is constant. The proportionality factor, as we have seen, is the same as in Planck's theory, though since it cancels out during the proof of the law of radiation, this fact is of no importance here.

Second, in Part III, Chapters III and IV, since the energy of a magneton can never get below the amount it has just after emission, the excess over this amount is what must be considered in the calculation of the entropy of the system.

Third, in the calculation of the distribution of energy among the oscillators, Part IV, Chapter III, we must divide them into two classes, those which are not emitting at the instant in question, and those that are; for the former, Planck's argument on the fraction of them in any energy interval at any time applies without change. Then since each one emits exactly as much energy as it absorbs, and at a constant rate, the distribution of the emitting ones, and therefore of both classes together, must be exactly like that of the absorbing ones. The transfer of energy to and from the molecular motions, being independent of the amount of energy in the steady current, cannot affect this distribution.

Thus we have Planck's law, derived by an argument almost identical with his, from assumptions which, although they are not so simple as his, are inconsistent with the classical dynamics only in the internal structure of the magneton. Being consistent with the classical electrodynamics, they correlate the phenomena of heat radiation with those of optics and electricity, at the same time giving an account of the laws of photo-electric and photo-chemical phenomena. Finally, being based on Parson's magneton, they correlate all these phenomena with those of chemical combinations.